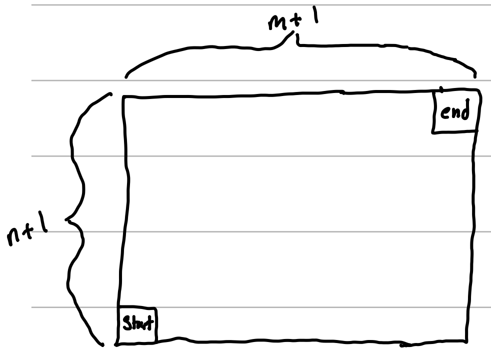
Lecture 20

- **R - combination**
  - Subsets
  - choosing r items from a pool of n items
  - order does <u>NOT</u> matter
- denoted $\binom{n}{r}$ → "n choose r"
- $\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$

Example



- you have to move n times upwards
- you have to move m times upwards
- So you have $n+m$ total moves.
- So the total paths $\binom{n+m}{m}$
- Also $\binom{m+n}{n} = \binom{m+n}{m}$

Example A fair coin tossed 20 times.

- How many sequences can we have? → 2^n → 2^{20}
- How many sequences contain exactly 2 heads? → $\binom{20}{2}$
- How many sequences contain at most 2 heads? → $\binom{20}{2} + \binom{20}{1} + \binom{20}{0}$
- How many of the sequences contain the same number of heads and tails → $\binom{20}{10}$

$\boxed{\text{Ex}}$ There are 20 distinct people. How many ways are there to divide them into two identical-sized groups?

↳ Group 1 × Group 2

↳ $\binom{20}{10} \times \binom{10}{10}$

↳ but this over counts because $P_1 \cdots P_{10}$ in G1 is same as $P_1 \cdots P_{10}$ in G2

↳ So 2 cases count as the same

↳ so final answer $\dfrac{\binom{20}{10} \cdot \binom{10}{10}}{2!}$ via k-to-1 rule

$\boxed{\text{Example}}$ 52 cards in a deck, dealer distributes 5 cards to each of 4 players. (Player A first, then B, C, D)

- How many ways to distribute cards (what players get what cards matters)?

$$\begin{array}{ccccccc} A & & B & & C & & D \\ \binom{52}{5} & \cdot & \binom{47}{5} & \cdot & \binom{42}{5} & \cdot & \binom{37}{5} \end{array} = \text{final answer}$$

- If Groups are not differentiated then:

$$\frac{\binom{52}{5} \cdot \binom{47}{5} \cdot \binom{42}{5} \cdot \binom{37}{5}}{4!} = \text{final answer (via } k \text{ to 1 rule)}$$

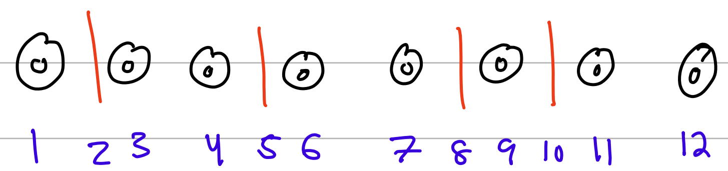
- Optional: $\binom{n}{r}$ is also called <u>binomial coefficent</u>.

Counting by Compliment

- We have n people but person A doesn't want to stand next to Person B.
- So the cases where A and B (A to the left of B) stand together is $(n-1)!$
- So the cases where A and B (B to the left of A) stand together is $(n-1)!$
- So A and B stand together a total of $2(n-1)!$ times.
- So answer is $n! - [2(n-1)!]$ via count by compliment

R-combination with Repetition (or multiset)

- There are 5 flavors of Donuts.
- I want to buy 8 donuts ≡ I want to find a multiset of 8 donuts with repeated flavors.



- Arbitrarily places bars
- There are 12 spots
  - ↳ So how many ways can you pick?
    - ↳ $\binom{12}{4}$